SUSX-TH-99-013

Destabilizing the gaugino condensate in modular invariant supergravity models

Thomas Dent*

Centre for Theoretical Physics, University of Sussex, Brighton BN1 9QH, U.K.

September 1999

Abstract

We investigate the stability of the hidden sector gaugino condensate in a SL(2,**Z**)-invariant supergravity model inspired by the $E_8 \otimes E_8$ heterotic string, using the effective chiral superfield formalism. We calculate the Planck-suppressed corrections to the "truncated approximation" for the condensate value and the scalar potential. A transition to a phase with zero condensate occurs near special points in moduli space and at large compactification radius. We discuss the implications for the T-modulus dependence of supersymmetry-breaking.

Dynamical supersymmetry-breaking via gaugino condensation [1] in a hidden sector is a major component of realistic supergravity unified theories. In heterotic string theory with gauge group $E_8 \otimes E_8$, the second E_8 factor provides a suitable hidden sector [2] with one or more confining gauge groups (depending on the details of symmetry-breaking). Assuming weak coupling at unification, the gauge coupling will become strong at an energy scale[1] $\Lambda \sim e^{-1/2bg_X^2}$, where g_X is the unified gauge coupling and b is the one-loop beta function coefficient defined so that $\beta(g) = -bg^3 + \ldots$. The vacuum expectation value of the gaugino bilinear $\langle \mathrm{Tr}\lambda^\alpha \lambda_\alpha \rangle$ switches on with a value of the order of Λ^3, breaking local supersymmetry. Supersymmetry-breaking is mediated by the dilaton and moduli superfields S, T_i in the four-dimensional supergravity effective theory: the auxiliary fields F^S and F^{T_i} take values of the order of $\langle \lambda\lambda \rangle$, the flat scalar potential for S and T_i is lifted and supersymmetry is broken softly in the visible sector [3, 4].

It has has usually been assumed that the condensate is well described by the *globally* supersymmetric gauge theory, since the confinement scale is well below M_P. In the limit of global supersymmetry $M_P \to \infty$, the condensate does not break supersymmetry [5, 6]: its value is determined in the effective superpotential approach [5], where the gaugino bilinear is the lowest component of the composite chiral superfield $U = \mathrm{Tr}W^\alpha W_\alpha$, by setting the auxiliary field $F_U = -(\partial \bar{W}_{np}/\partial \bar{U})$ to zero, where W_{np} is the nonperturbative superpotential generated by the gauge dynamics. The connection with supergravity models is via the value of the gauge coupling at unification, which is determined by the vacuum expectation values of the dilaton and moduli. The same condensate value follows from minimising the scalar potential in supergravity if higher-order terms in U are neglected [7, 8]: we call this the "truncated approximation".

*E-mail address: t.e.dent@sussex.ac.uk

[1]We use reduced Planck units with $\kappa^{-1} = 1/\sqrt{8\pi G} = 1$.

The resulting condensate value is substituted back into the nonperturbative superpotential, which is then a function of the dilaton and moduli only and serves as an effective source of supersymmetry-breaking [7, 9, 10].

If the supergravity corrections to the truncated approximation are significant, there will be effects on supersymmetry-breaking, the stabilization of the dilaton and moduli, and inflationary models which use gaugino condensation to provide a positive potential [11, 12]. The size of deviations from the truncated approximation can be found by minimising the supergravity scalar potential as a function of U. Lalak et al. [13] used a similar approach to take into account the backreaction of other fields on the condensate. Our analysis differs from theirs, since we assume a different mechanism for stabilizing the dilaton, we set all superpotential terms which do not depend on the condensate to zero, and we will be interested mainly in the effects on the compactification moduli of string theory, rather than the dilaton.

The vacuum expectation value (vev) of the gaugino bilinear is determined by the scalar component of U at the stationary point of the effective action $\Gamma(U,S,T_i)$ as defined by Burgess et al. [14], where U is treated as a *classical* field which represents the expectation value of the composite operator $\mathrm{Tr}(WW)$. In the simplest case of a single confining gauge group with no matter, the condensate is formally described by the field $U \equiv \hat{U}/S_0^3$, where $\hat{U} = \langle \mathrm{Tr} W^\alpha W_\alpha \rangle$ and W^α is the gauge field strength chiral superfield. S_0, the chiral compensator superfield, is introduced to simplify the formulation of supergravity coupled to matter [15] in the superconformal tensor calculus[2]. The gaugino bilinear $\langle \mathrm{Tr}\lambda^\alpha \lambda_\alpha \rangle$ is then the lowest component of $\hat{U}$ ($\theta = \bar{\theta} = 0$).

The effective action $\Gamma(U,S,T_i)$ is constructed as a supergravity action, with a superpotential and Kähler potential for the condensate which can be found by considering the symmetries of the underlying theory and the corresponding anomalies [5, 14]. In the case of a single overall modulus, $T_1 = T_2 = T_3 \equiv T$, the superpotential is

$$W_{np}(U,S,T) = \frac{1}{4} f_{GK}(S,T) U + \frac{b}{6} U \ln(cU) \tag{1}$$

where $f_{GK}(S,T)$ is the gauge kinetic function, equal to S at tree level, which depends on the modulus T through string loop threshold corrections [16], and c is a constant which will be discussed shortly. The superpotential can be written as

$$\begin{aligned} W_{np}(U,S,T) &= \frac{b}{6} U \ln(cU\omega(S)h(T)) & (2) \\ &= \frac{b}{6} U \left(\ln\left(\frac{U}{\Lambda_c^3}\right) - 1 \right) & (3) \end{aligned}$$

where $\omega(S) \equiv e^{3S/2b}$ and $h(T)$ is a function with appropriate transformation properties under the target-space duality group SL(2,**Z**) [17, 18]. The condensation scale Λ_c is defined by (3) as

$$\Lambda_c = e^{-S/2b - 1/3} (ch(T))^{-1/3}.$$

Changing the constant c is equivalent to changing the unified gauge coupling or the beta-function; the value of c could be determined, if f_{GK} is known, by comparing the resulting condensation scale with the vev of $\mathrm{Tr}(W^\alpha W_\alpha)$ derived from instanton calculations in global SUSY [19](compare [17]).

[2]The components of S_0 are determined by gauge-fixing the superconformal symmetries so that the Einstein term in the Lagrangian is canonically normalised [14].

The Kähler potential was determined in [14] as

$$K(U,S,T) = \tilde{K} - 3\ln\left(1 - \frac{9}{\gamma}e^{\tilde{K}/3}(U\bar{U})^{1/3}\right) \tag{4}$$

where

$$\tilde{K} = K_p(S,T) = -\ln(S+\bar{S}) - 3\ln(T+\bar{T}) \tag{5}$$

is the perturbative string tree-level Kähler potential, and γ is an real constant of order 1; we will show that the value of γ does not affect our results. The expression (5) also holds at one loop if the Green-Schwarz anomaly cancellation coefficients δ_{GS} are neglected: we set $\delta_{GS} = 0$ for simplicity. The expression (5) can be generalised to include stringy nonperturbative effects [20] which have been invoked in order to stabilise the dilaton [21]: we replace (5) by

$$\tilde{K} = P(y) - 3\ln(T+\bar{T}).$$

where $P(y)$ is a real function of $y \equiv (S+\bar{S})$. The dilaton dynamics enter only through the auxiliary field F^S, so we do not need to specify the form of $P(y)$; however, we require $P''(y) > 0$ for the kinetic term for the dilaton to have the right sign.

Note that K becomes ill-defined for $(9/\gamma)e^{\tilde{K}/3}(U\bar{U})^{1/3} \to 1$. It is not surprising that the effective action for U breaks down in this limit, since it means that the condensate forms at the string scale and the gauge group is strongly-coupled *at unification*. This limitation has implications for the behaviour of the condensate at large $\mathrm{Re}(T)$, which will be discussed later.

The supergravity action is invariant under target-space SL(2,$\mathbf{Z}$) transformations [22, 23] if the superpotential transforms as a modular form[3] of weight -3. The modulus T transforms as

$$T \to \frac{aT - ib}{icT + d} \tag{6}$$

where a, b, c, d are integers satisfying $ad - bc = 1$. Then U must transform as

$$U \to \zeta_U (icT+d)^{-3} U, \tag{7}$$

where ζ_U is a (field-independent) phase factor; also, the function $h(T)$ transforms as $h(T) \to \zeta_U^{-1}(icT+d)^3 h(T)$. The U-dependent part of the Kähler potential is then modular invariant. In order to avoid singularities inside the fundamental domain of SL(2,$\mathbf{Z}$), $h(T)$ must be of the form

$$h(T) = \eta^6(T)/H \tag{8}$$

where $\eta(T)$ is the Dedekind eta function and H is a constant or a modular invariant function of T without singularities [10].

The part of $\Gamma(U,S,T)$ relevant to finding the value of the condensate is the scalar potential, which is given as usual by

$$V = e^G (G_i (G^{-1})^i_j G^j - 3)$$

where i and j range over the scalar components of U, S and T, $G^i \equiv \partial G/\partial\phi_i$, $G_i \equiv \partial G/\partial\bar{\phi}^i$ and G^{-1} is defined by $(G^{-1})^i_j G^j_k = \delta^i_k$. We find, as in [18, 9]:

$$V = \left(\frac{b}{6}\right)^2 \frac{|z|^4}{(9/\gamma)^2(1-|z|^2)^2}\left\{3\left|1 + \ln(c\omega(S)h(T)e^{-\tilde{K}/2}z^3)\right|^2 + C_2|z|^2\right\} \tag{9}$$

[3]For a discussion of modular forms see [24] or the appendix of [10].

where

$$z = \frac{3}{\sqrt{\gamma}}(e^{\tilde{K}/2}U)^{1/3} = \frac{3}{\sqrt{\gamma}}\, e^{P(y)/6}(T+\bar{T})^{-1/2}U^{1/3},$$

$$C_2 = \frac{1}{P''(y)}\left|P'(y) - \frac{\omega'(S)}{\omega(S)}\right|^2 + \frac{1}{3}\left|3 + (T+\bar{T})\frac{h'(T)}{h(T)}\right|^2 - 3, \tag{10}$$

and we have absorbed a factor of $(9/\gamma)^{-3/2}$ into the constant c. The only dependence on γ of the potential (9) is in the overall scale, which has no effect on the existence or stability of minima, so we set $9/\gamma = 1$ from now on.

Note that the potential depends on the phase of z only through the first term inside the brackets. If we hold $|z|$ fixed and vary the phase, a minimum can only occur when the argument of the logarithm $(c\omega(S)h(T)e^{-\tilde{K}/3}z^3)$ is real and positive. The condensate phase $\arg(z)$ is then aligned with $\arg(h(T)^{-1/3})$ [17], assuming that c and $\omega(S)$ are real. The dependence of the potential on $|z|$ is then determined by two real parameters, $|c\omega(S)h(T)|e^{-\tilde{K}/2}$ and C_2.

The truncated approximation follows from minimising (9) in the rigid supersymmetry limit where $M_P \to \infty$, $\omega(S) \to \infty$, $z \to 0$ and C_2 is held constant. The first term inside the bracket completely dominates the dependence on z and there is a zero-value minimum with unbroken supersymmetry satisfying $\partial W/\partial U = 0$ at

$$z_{tr} = e^{\tilde{K}/6}\Lambda_c = \frac{e^{P(y)/6-1/3}}{(T+\bar{T})^{1/2}}(c\omega(S)h(T))^{-1/3}. \tag{11}$$

Substituting this value back into (2) leads directly to the well-known form of nonperturbative superpotential [25, 10, 9]

$$W_{np}(S,T) = \frac{b}{6}\Lambda_c^3 \propto \frac{\omega^{-1}(S)H}{\eta^6(T)} \tag{12}$$

which has been argued to occur independently of any particular supersymmetry-breaking mechanism [22, 10].

Returning to local supersymmetry, when $C_2|z|^2$ is non-zero, $z = z_{tr}$ is not a minimum of the scalar potential and we cannot invoke the condition that the hidden sector gauge dynamics preserve supersymmetry. For small $C_2|z|^2$, the truncated value is a good approximation; however, we would like to find the size of corrections, and to investigate the behaviour of the potential when the truncated approximation fails.

Figure 1 shows the effect on the potential as a function on $|z|$ of varying C_2 while keeping $|z_{tr}|$ fixed. For $-3 < C_2 < 0$ there is an absolute minimum with $|z|$ near the truncated value and negative vacuum energy. For C_2 small and positive there is a local minimum at $|z|$ near $|z_{tr}|$ and the vacuum energy is positive; as C_2 increases the minimum becomes shallower and moves towards smaller values of $|z|$; finally the minimum merges with a point of inflection and the condensate value goes discontinuously to zero, as in a first-order phase transition[4].

The stationary point condition for the potential (9) is a transcendental equation in z, so we use a series expansion for z near the truncated value and also search for stationary points numerically. To model the behaviour of the potential we write it as

$$V(|z|) = \left(\frac{b}{6}\right)^2 \frac{|z|^4}{(1-|z|^2)^2}\left\{3\left(\log\left|\frac{z}{z_{tr}}\right|^3\right)^2 + C_2|z|^2\right\}. \tag{13}$$

[4]It was already noticed by Ferrara et al. [17] that for some values of parameters the only stationary point of the potential is at $z = 0$.

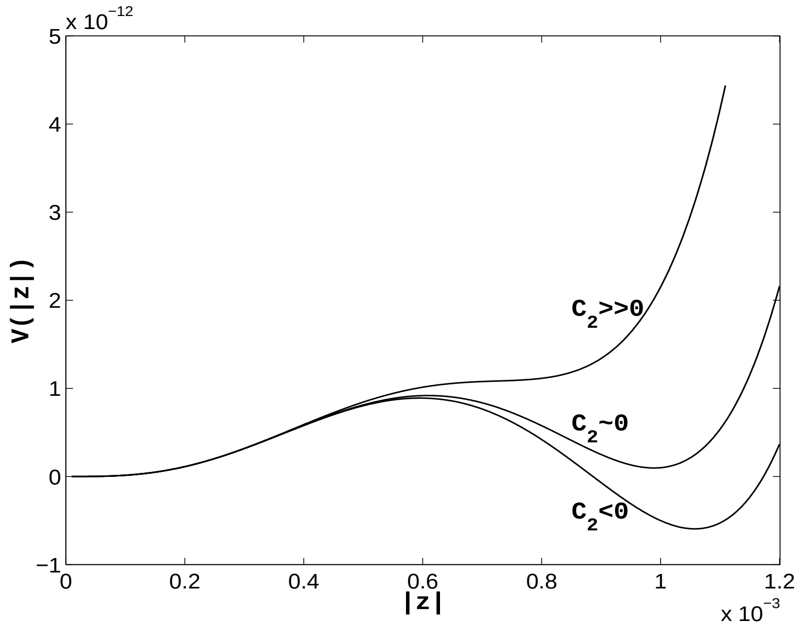

Figure 1: Effect of varying the parameter C_2 on the potential $V(|z|)$, setting $|z_{tr}| = 10^{-3}$.

and expand to third order in $\Delta z \equiv |z| - |z_{tr}|$, obtaining

$$V = \left(\frac{b}{6}\right)^2 \frac{|z_{tr}|^4}{(1-|z_{tr}|^2)^2} \left(C_2|z_{tr}|^2 + a_1\Delta z + a_2\Delta z^2 + a_3\Delta z^3 + \dots\right) \tag{14}$$

where

$$\begin{aligned} a_1 &= 2C_2|z_{tr}|(1-|z_{tr}|^2)(3-|z_{tr}|^2) \\ a_2 &= 27\frac{(1-|z_{tr}|^2)}{|z_{tr}|^2} + C_2(15 - 4|z_{tr}|^2 + |z_{tr}|^4) \\ a_3 &= 27\frac{(1-|z_{tr}|^2)(3+|z_{tr}|^2)}{|z_{tr}|^3} + 4C_2\frac{(5+3|z_{tr}|^2)}{|z_{tr}|(1-|z_{tr}|^2)}. \end{aligned}$$

Since $|z_{tr}|$ is $\mathcal{O}(10^{-1})$ or smaller in most cases of physical interest, we neglect $|z_{tr}|^2$ next to 1 and take

$$a_1 = 6C_2|z_{tr}|,\ a_2 = \frac{27}{|z_{tr}|^2} + 15C_2,\ a_3 = \frac{81}{|z_{tr}|^3} + \frac{20C_2}{|z_{tr}|}. \tag{15}$$

The series expansion (14) has stationary points if $a_2^2 \geq 3a_1a_3$, with a stationary point of inflection if the equality is satisfied. In terms of the parameter $x \equiv C_2|z_{tr}|^2$, the condition for a minimum to exist is

$$27 - 24x - 5x^2 > 0 \tag{16}$$

which is solved, for positive x, by $x < 0.94$.

Where (16) holds, the position of the minimum is given by

$$\Delta z_{min} = \frac{1}{a_3}\left(-a_2 + \frac{2}{3}\sqrt{a_2^2 - 3a_1a_3}\right) = |z_{tr}|\frac{\left(-9 - 5x + \sqrt{81 - 72x - 15x^2}\right)}{81 + 20x} \tag{17}$$

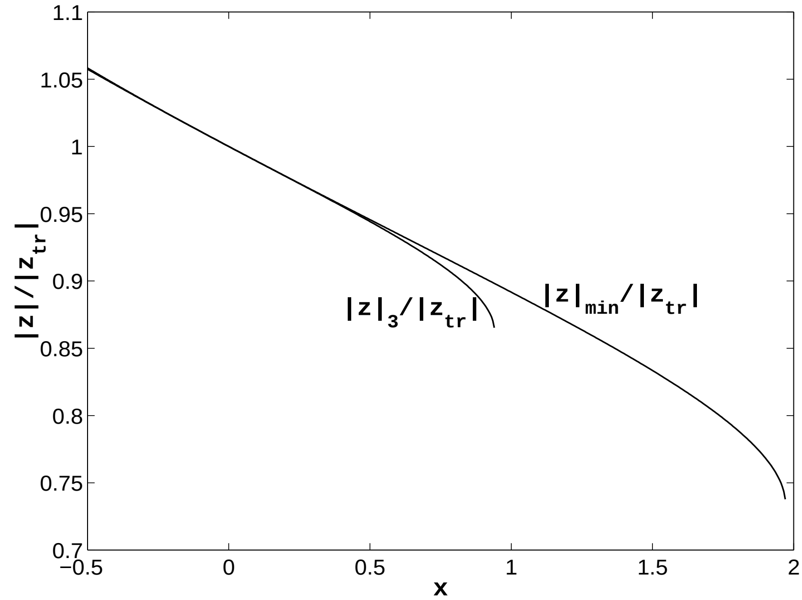

Figure 2: Values of $|z|$ at the minimum as a function of $x \equiv C_2|z_{tr}|^2$, scaled by $|z_{tr}| = 10^{-3}$. $|z|_{min}$ and $|z|_3$ minimise the full potential (13) and the third-order series expansion, respectively.

which can be expanded as $\Delta z_{min} = |z_{tr}|(-x/9 + x^2/162 + \ldots)$. The leading order corrections to the minimum value of the potential in the truncated approximation are

$$V(|z|_3) - V(z_{tr}) = \frac{b^2}{36}\frac{|z_{tr}|^4}{(1-|z_{tr}|^2)^2}\left(-\frac{x^2}{3} + \frac{2x^3}{27} + \ldots\right) = V(z_{tr})\left(-\frac{x}{3} + \frac{2x^2}{27} + \ldots\right)$$

where $|z|_3 = |z_{tr}| + \Delta z_{min}$.

We also found the condensate value $|z|_{min}$ at the minimum of the full potential (13) numerically, as a function of x. For all values of $|z_{tr}| < 2 \times 10^{-2}$, the phase transition at which $|z|_{min}$ goes to zero occurs for x between 1.968 and 1.972. For larger $|z_{tr}|$, the transition occurs at slightly smaller values of x, indicating a dependence on higher-order terms in $|z_{tr}|$: for example, at $z_{tr} = 0.2$ the transition occurs for $1.872 < x < 1.876$. Figure 2 shows the value of $|z|/|z_{tr}|$ at the minimum for the full potential and for the series expansion (14), as a function of x. We take $|z_{tr}| = 10^{-3}$; however, the results are very similar for all values of $|z_{tr}|$ below about 0.3. Both curves deviate significantly from 1, and the series expansion is a good approximation to the true minimum for $x < 0.7$. While the series expansion gives a qualitative picture of the behaviour of $|z|$ near the phase transition, the value of x where the transition occurs is about twice that predicted by (16). This is not surprising, since Δz is large here and the series expansion is less accurate.

Figure 3 shows the value of the potential at its minimum and at the truncated value of z, and the minimum value of the series expansion, as a function of x. Again, there is a significant deviation from the truncated result for $x >$ few$\times 0.1$.

We now discuss in what physical situations the corrections to the truncated approximation may be significant. In order to generate phenomenologially reasonable supersymmetry breaking in the visible sector, we require the gravitino mass $m_{3/2}$ to be at the TeV scale (see for example [3]); for a single condensate it is given by $m_{3/2} \simeq \frac{b}{6}|z|^3$, which implies a vev of $|z|_0 = \text{few} \times 10^{-5}$ for the physical values taken by the dilaton and moduli today. The parameter $x = C_2|z_{tr}|^2$

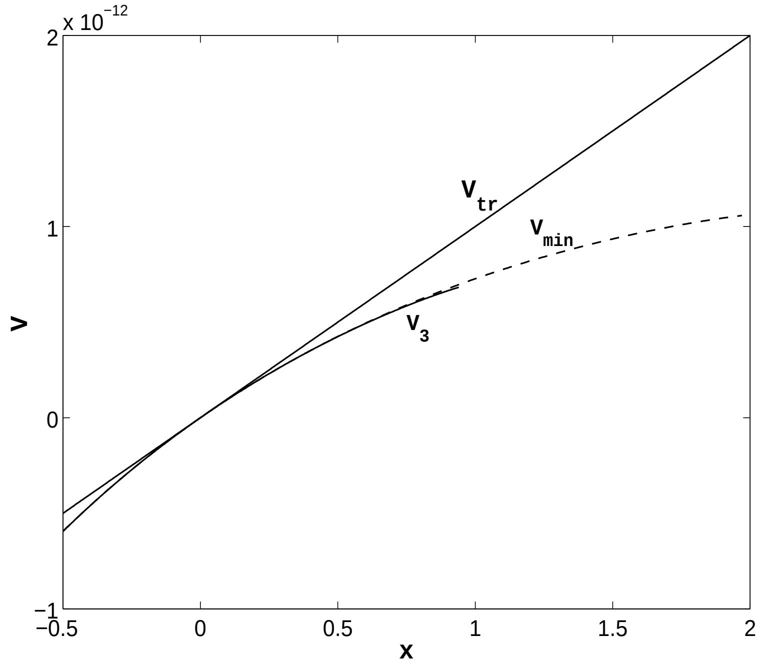

Figure 3: V_{tr} and V_{min} are the value of the potential at $|z_{tr}|$ and at $|z|_{min}$, respectively. V_3 is the value of the third-order series expansion (14) at its minimum $|z|_3$ ($|z_{tr}| = 10^{-3}$).

should be at least $\mathcal{O}(0.1)$ for corrections to the truncated approximation to be appreciable, so unless one of the terms in C_2 is of order 10^9 or larger the truncated approximation is not in any danger today.

The first term in (10) can become large only if the ratio $P'(y)^2/P''(y)$ does (note that ω'/ω is a constant of order 10). It is unlikely that the function $P(y)$, which describes stringy non-perturbative effects on the dilaton, will give rise to singularities or very large numbers; also, since $P''(y)$ must be positive, P'' cannot be very small without fine-tuning. The only limit in which the dilaton-dependence causes the truncated approximation to break down is at strong coupling ($S \to 0$), where the effective action $\Gamma(U, S, T_i)$ is anyway not valid.

The second term involving $h'(T)/h(T)$, where $h(T)$ is given by (8), may become large if $H(T)$ is zero or singular for any value of T in the fundamental domain; in addition, the eta function decays exponentially with T at $\mathrm{Re}(T) \to \infty$, which leads to a divergence at large $\mathrm{Re}(T)$[5]. Nothing can be said within the effective theory about the limit of large $\mathrm{Re}(T)$ or the singularities of $H(T)$, since the condensation scale Λ_c diverges at these points. However, at zeros of $H(T)$, f_{GK} becomes large and positive and the condensation scale goes to zero, so the effective action can be used to describe the condensate at these points.

Consider the ansatz for the T-dependence of the superpotential (2,8) involving the absolute modular invariant $J(T)$ [10], where $H(T)$ takes the form

$$H(T) = (J - 1)^{m/2} J^{n/3} p(J) \tag{18}$$

where m, n are positive integers or zero and $p(J)$ is a polynomial in J. This form for H

[5]This behaviour can be understood in terms of string states charged under the gauge group which become light (mass $\ll M_P$) at certain values of T [10]. At large $\mathrm{Re}(T)$ the extra dimensions decompactify and an infinite number of Kaluza-Klein states become light, producing a linear divergence in the gauge kinetic function. At poles or zeros of H a finite number of states become light, leading to a logarithmic divergence in f_{GK}.

has no singularity in the fundamental domain, but may have zeros at the "fixed point" values $T = \rho = e^{i\pi/6}$ and $T = 1$, since $J \propto (T-\rho)^3$ near $T = \rho$ and $J \propto (T-1)^2$ near $T = 1$.

As T approaches a zero of $H(T)$ (we consider $T = \rho$ for definiteness), $|z_{tr}|$ vanishes with $|T-\rho|^{n/3}$ and C_2 diverges with $|T-\rho|^{-2}$. The potential in the truncated approximation $V_{tr} = (b/6)^2|z_{tr}|^6 C_2$ varies smoothly as $T \to \rho$; in the case $n = 1$, the potential tends to a non-zero value, even though the condensate vanishes at $T = \rho$ and supersymmetry is unbroken. This puzzle is resolved by looking at the stabilization of the condensate explicitly: as $T \to \rho$, the parameter x diverges as $x \propto |T-\rho|^{2n/3-2}$, so for $1 \leq n < 3$ the truncated approximation breaks down and $|z|_{min}$ goes to zero abruptly at some finite, small value of $|T-\rho|$. The scalar potential as a function of T develops a "hole" around $T = \rho$, inside which the condensate and scalar potential vanish and supersymmetry is restored. If T were to fall into such a region, since $z = 0$ is always a stationary point of the effective action, a non-zero condensate would not immediately be restored if T then rolled out to a value where (16) held: the universe would have to "tunnel" back to a supersymmetry-breaking vacuum.

We also consider varying T away from its present-day value in a perturbative heterotic string vacuum with $S \simeq 2$ and $T_0 = \mathcal{O}(1)$, to find the effect of our treatment of the condensate on the (cosmological) evolution of the moduli if $\mathrm{Re}(T)$ becomes large. As was argued above, the present-day value of the condensate should be $|z|_0 \simeq 2 \times 10^{-5}$ in Planck units. In the case where $h(T) = \eta^6(T)$, z_{tr} varies as $\eta^{-2}(T)$: for $\mathrm{Re}(T) > 3$ this is well approximated by $\eta^{-2}(T) = e^{\pi T/6}$, while the relevant part of C_2 is given by $C_2 \simeq \pi^2(T+\bar{T})/12 \simeq 1.64\ \mathrm{Re}(T)$. The effective theory breaks down when $|z_{tr}|$ approaches 1, which occurs when $T \simeq 21$; however, a non-zero condensate is only stable for $1.64\, T e^{\pi T/3}(2 \times 10^{-5})^2 < 1$ (see eqn. 16), which implies that $T < 17$–19 (depending on the exact values of $|z_{tr}|_0$ and $\eta(T_0)$). At the phase transition where the condensate collapses, $C_2 \simeq 30$ so $|z_{tr}| \simeq 0.18$, which is consistent with using the effective action.

This analysis can also be applied to models of cosmological inflation which use the T modulus to provide a positive vacuum energy [11, 12]. In the region of the complex T plane where inflation occurs, $V \simeq |z_{tr}|^6 C_2$ should be of order $10^{-10}\kappa^{-4}$ to produce the correct amplitude of CMB fluctuations at large angular scales [12]. Typically, this can be achieved for $T = \mathcal{O}(1)$ and $C_2 = \mathcal{O}(10)$, implying that $|z_{tr}|$ is of order 10^{-2} and $x \sim 10^{-3}$: the condensate scale here is much larger than that required to give realistic SUSY-breaking. Repeating the previous analysis, we find that the effective theory becomes invalid at $\mathrm{Re}(T) \simeq 9$ and a non-zero condensate ceases to be stable for $\mathrm{Re}(T)$ between 6.5 and 7.

One might hope to find a form of T-dependent superpotential (8) involving $J(T)$ (18) which allows T to take large values while keeping modular invariance of the effective action, since it has been suggested that a realistic vacuum of the strongly-coupled heterotic string [26] may have $\mathrm{Re}(T)$ of order 25 [27]. Unfortunately the first term in the large T expansion of $J(T)$ increases as $e^{2\pi T}$, so the exponential growth of $z_{tr} \propto H^{1/3}\eta^{-2}$ at large $\mathrm{Re}(T)$ is likely to be faster than for constant H, and there is little prospect of stabilizing either the condensate or T with this ansatz.

In conclusion, we studied the stability of a non-zero gaugino condensate in an effective supergravity model motivated from compactifications of heterotic string theory with duality group SL(2,**Z**). We defined a parameter x which determines the deviation of the value of the condensate from its value in global supersymmetry. Using a series expansion for the effective potential, the corrections to the truncated approximation at small x were obtained to second order in $(\Lambda_c/M_P)^2$ for the value of the condensate, and to first order in $(\Lambda_c/M_P)^2$ for the vacuum energy as a function of the dilaton and T modulus. Numerical minimisation of the effective potential confirms the series expansion results at small x, and reveals a phase transition at large x where the condensate vanishes discontinuously. This appears to be caused by the backreaction of the dilaton and T modulus on the hidden sector gauge dynamics. There are implications for the T-dependence of hidden sector supersymmetry-breaking : the auxiliary field F^T and the scalar potential receive large corrections or go to zero abruptly near special points in moduli space and at large $\mathrm{Re}(T)$. In particular, T is not stabilized against arbitrarily large fluctuations.

It would be interesting to find the effect of non-zero Green-Schwarz anomaly cancellation coefficients on the S- and T- dependence of the scalar potential, and also to analyse the stability of the condensate in the linear supermultiplet formalism [28]. If the potential for T is very flat near the minimum, it may be worth investigating the effect of including Planck-suppressed corrections on the T-dependent Yukawa couplings and soft supersymmetry-breaking terms, particularly if CP-violating phases depend strongly on the vev of T [29].

Thanks are due to David Bailin for suggesting the problem, to Jackie Grant for help with the figures and to Malcolm Fairbairn for a critical reading of the paper in its early stages. TD is supported by PPARC studentship PPA/S/S/1997/02555.

References

[1] H.P. Nilles, Phys. Lett. B 115 (1982) 193; S. Ferrara, L. Girardello and H.P. Nilles, Phys. Lett. B 125 (1983) 457.

[2] J.P. Derendinger, L.E. Ibáñez and H.P. Nilles, Phys. Lett. B 155 (1985) 65; M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. B 156 (1985) 55; T.R. Taylor, Phys. Lett. B 164 (1985).

[3] B. de Carlos, J.A. Casas and C. Muñoz, Nucl. Phys. B 399 (1993) 623.

[4] V. Kaplunovsky and J. Louis, Phys. Lett. B 306 (1993) 269; A. Brignole, L.E. Ibáñez and C. Muñoz, Nucl. Phys. B 422 (1994) 125.

[5] G. Veneziano and S. Yankielowicz, Phys. Lett. B 113 (1982) 231.

[6] E. Witten, Nucl. Phys. B 202 (1982) 253.

[7] J.A. Casas, Z. Lalak, C. Muñoz and G.G. Ross, Nucl. Phys. B 347 (1990) 243.

[8] D. Lüst and T.R. Taylor, Phys. Lett. B 253 (1991) 335.

[9] B. de Carlos, J.A. Casas and C. Muñoz, Phys. Lett. B 263 (1991) 248.

[10] M. Cvetič, A. Font, L.E. Ibáñez, D. Lüst and F. Quevedo, Nucl. Phys. B 361 (1991) 194.

[11] D. Bailin, G. Kraniotis and A. Love, Phys. Lett. B 443 (1998) 111.

[12] S. Thomas, Phys. Lett. B 351 (1995) 424; T. Banks, M. Berkooz, G. Moore, S.H. Shenker and P.J. Steinhardt, Phys. Rev. D52 (1995) 3548.

[13] Z. Lalak, A. Niemayer and H.P. Nilles, Nucl. Phys. B 453 (1995) 100.

[14] C.P. Burgess, J.P. Derendinger, F. Quevedo and M. Quiros, Ann. Phys. (N. Y.) 250 (1996) 193.

[15] S. Ferrara, L. Girardello, T. Kugo and A. Van Proeyen, Nucl. Phys. B 223 (1983) 191.

[16] L.J. Dixon, V. Kaplunovsky and J. Louis, Nucl. Phys. B 355 (1991) 649.

[17] S. Ferrara, N. Magnoli, T.R. Taylor and G. Veneziano, Phys. Lett. B 245 (1990) 409.

[18] T.R. Taylor, Phys. Lett. B 252 (1990) 59.

[19] D. Amati, K. Konishi, Y. Meurice, G.C.Rossi and G. Veneziano, Phys. Rep. 162 (1988) 169.

[20] S.H. Shenker, in Cargèse 90 Proceedings, Random surfaces and quantum gravity, Cargèse (France).

[21] T. Banks and M. Dine, Phys. Rev. D 50 (1994) 7454; J.A. Casas, Phys. Lett. B 384 (1996) 103; P. Binétruy, M.K. Galliard and Y.-Y. Wu, Nucl. Phys. B 481 109; K. Choi, H.B. Kim and H. Kim, Mod. Phys. Lett. A14 (1999) 125.

[22] S. Ferrara, D. Lüst, A. Shapere and S. Theisen, Phys. Lett. B 225 (1989) 363.

[23] S. Ferrara, D. Lüst and S. Theisen, Phys. Lett. B 233 (1989) 147.

[24] R. Rankin, Modular forms and functions (C.U.P., Cambridge, 1977).

[25] A. Font, L.E. Ibáñez, D. Lüst and F. Quevedo, Phys. Lett. B 245 (1990) 401.

[26] P. Hořava and E. Witten, Nucl. Phys. B 460 (1996) 506; E. Witten, Nucl. Phys. B 471 (1996) 135.

[27] T. Banks and M. Dine, Nucl. Phys. B 479 (1996) 173: see also the last reference of [21].

[28] P. Binétruy, M.K. Galliard and Y.-Y. Wu, Nucl. Phys. B 493 (1997) 27.

[29] D. Bailin, G.V. Kraniotis and A. Love, Nucl. Phys. B 518 (1998) 92, Phys. Lett. B 435 (1998) 323.